HARMONIC MEAN

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Harmonic Mean Definition

The Harmonic Mean (HM) is defined as the reciprocal of the arithmetic mean of the given data values. It is based on all the observations, and it is rigidly defined. Harmonic mean gives less weightage to the large values and large weightage to the small values to balance the values properly. In general, the harmonic mean is used when there is a necessity to give greater weight to the smaller items. It is applied in the case of times and average rates.

Harmonic Mean Formula

Since the harmonic mean is the reciprocal of the arithmetic mean, the formula to define the harmonic mean "H" is given as follows:

If $x_1, x_2, x_3, \ldots, x_n$ are the individual items up to n terms, then,

Harmonic Mean, $H = n / [(1/x_1)+(1/x_2)+(1/x_3)+\ldots+(1/x_n)]$

Relationship Between Arithmetic Mean, Geometric Mean and Harmonic Mean

The three means such as arithmetic mean, geometric mean, harmonic means are known as Pythagorean means. The formulas for three different types of means are:

Arithmetic Mean = $(a_1 + a_2 + a_3 + \ldots..+a_n) / n$

Harmonic Mean = $n / [(1/a_1)+(1/a_2)+(1/a_3)+\ldots+(1/a_n)]$

Geometric Mean = a1.a2.a3…an−−−−−−−−−−−−√n

If G is the geometric mean, H is the harmonic mean, and A is the arithmetic mean, then the relationship joining them is given by

G=AH−−−√

Weighted Harmonic Mean

Calculating weighted harmonic mean is similar to the simple harmonic mean. It is a special case of harmonic mean where all the weights are equal to 1. If the set of weights such as $w_1, w_2, w_3, \ldots, w_n$ connected with the sample space $x_1, x_2, x_3, \ldots., x_n$, then the weighted harmonic mean is defined by

HMw=∑ni=1wi∑ni=1wixi

If the frequencies “f” is supposed to be the weights “w”, then the harmonic mean is calculated as follows:

If x_1, x_2, x_3,…., x_n are n items with corresponding frequencies f_1, f_2, f_3, …., f_n, then the weighted harmonic mean is

$HM_w = N / [(f_1/x_1) + (f_2/x_2) + (f_3/x_3) + \ldots.(f_n/x_n)]$

Note:

1. f values are considered as weights
2. For continuous series, mid-value = (Lower limit + Upper limit)/2 is taken as x

Harmonic Mean Uses

The main uses of harmonic means are as follows:

- The harmonic mean is applied in the finance to the average multiples like the price-earnings ratio.
- It is also used by the market technicians in order to determine the patterns like the Fibonacci Sequences.
- In population genetics, the harmonic mean is used when calculating the effects of fluctuations in the census population size on the effective population size. The harmonic mean takes into account the fact that events such as population bottleneck increase the rate genetic drift and reduce the amount of genetic variation in the population. This is a result of the fact that following a bottleneck very few individuals contribute to the gene pool limiting the genetic variation present in the population for many generations to come.

- In chemistry and nuclear physics the average mass per particle of a mixture consisting of different species (e.g., molecules or isotopes) is given by the harmonic mean of the individual species' masses weighted by their respective mass fraction.

Merits and Demerits of Harmonic Mean

The following are the merits of the harmonic mean:

- It is rigidly confined.
- It is based on all the views of a series. It means that it cannot be computed by ignoring any item of a series.
- It is able to advance the algebraic method.

- It provides a more reliable result when the results to be achieved are the same for the various means adopted.
- It provides the highest weight to the smallest item of a series.
- It can be measured also when a series holds any negative value.
- It produces a skewed distribution a normal one.
- It produces a curve straighter than that of the A.M and G.M.

The demerits of the harmonic series are as follows:

- The harmonic mean is greatly affected by the values of the extreme items
- It cannot be able to calculate if any of the items is zero
- The calculation of the harmonic mean is cumbersome, as it involves the calculation using the reciprocals of the number.

<u>Example</u>

<u>#1</u>

The given distribution is grouped data and the variable involved is the ages of first year students, while the number of students represents frequencies.

Ages (Years) x	Number of Students f	$\frac{1}{x}$
13	2	0.1538
14	5	0.3571
15	13	0.8667
16	7	0.4375
17	3	0.1765
Total	$\sum f = 30$	$\sum\left(\frac{f}{x}\right) = 1.9916$

Now we will find the harmonic mean as

$\overline{X} = \frac{\sum f}{\sum\left(\frac{f}{x}\right)} = \frac{30}{1.9916} = 15.0631 \approx 15$ years.

#2

Example: Find the harmonic mean of the following data {8, 9, 6, 11, 10, 5} ?

Solution:

Given data: {8, 9, 6, 11, 10, 5}

So Harmonic mean = $\frac{6}{\frac{1}{8}+\frac{1}{9}+\frac{1}{6}+\frac{1}{11}+\frac{1}{10}+\frac{1}{5}}$

H = $\frac{6}{0.7936}$ = 7.560

Harmonic mean(H) = 7.560

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